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16. ABSTRACT

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For the two categories of entrainment studied, graphs are presented showing how the maximum time-averaged near-surface concentration in the horizontally homogeneous situation depends on averaging time and upon the angle between the wind velocity inside the mixed layer and just above.

In the steady lake-breeze situation the maximum mean value found in the laboratory is an upper limit to the maximum time-averaged value inland of the shoreline because of the absence of downstream diffusion in the laboratory experiments.

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GROUND-LEVEL CONCENTRATIONS DUE TO FUMIGATION INTO AN ENTRAINING MIXED LAYER

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Abstract—Laboratory measurements on fumigation into a convectively mixed layer indicate how the near surface concentration behaves with time. For a fumigating ribbon with initial dimensions much smaller than the mixed-layer depth, the near-surface concentrations are shown to be independent of both the initial lateral and vertical dimensions of the fumigating plume. Because of extensive irregularities in the local mixed-layer height, even in horizontally homogeneous conditions, a surprisingly long time is required between the onset and completion of fumigant entrainment.

For two categories of entrainment rate: slow and fast, the experiments indicate how the ensemble-averaged concentration near the surface reaches a greater maximum in less time for the more rapid entrainment.

A model which approximates the laboratory results is found to be grossly in error unless a very substantial time is allowed for vertical mixing of an entrained parcel of fumigant inside the mixed layer to be completed. The model and laboratory data together also suggest that horizontal mixing of entrained fumigants proceeds more slowly when the entrainment is more rapid.

For the two categories of entrainment studied, graphs are presented showing how the maximum time-averaged near-surface concentration in the horizontally homogeneous situation depends on averaging time and upon the angle between the wind velocity inside the mixed layer and just above.

In the steady lake-breeze situation the maximum mean value found in the laboratory is an upper limit to the maximum time-averaged value inland of the shoreline because of the absence of downstream diffusion in the laboratory experiments.

1. INTRODUCTION

The process of fumigation refers to the incorporation of contaminants initially lying above a turbulent boundary layer into the latter as it intercepts them. They are then diffused downward within the boundary layer, producing ground-level concentrations that may be noxious.

Two conceptually distinct cases in which fumigation can occur are:

(a) the boundary layer whose top at $z = \bar{h}(t)$ is horizontally homogeneous, in the mean, so that the mean depth $\bar{h}$ grows with time to intercept the fumigant and

(b) the steady state but sloping boundary-layer interface; e.g., just inland of a lake–land boundary where $\bar{h} = \bar{h}(x)$ and the entrainment occurs at a rate which just balances the horizontal advection of $\bar{h}$ along x by the mean wind, U.

In either case, the problem addressed here is one of determining, through laboratory measurements, the maximum average ground-level concentration measured over a time period τ due to fumigation into a convectively mixed layer whose entrainment or growth rate is w_e. The plume dimensions at the initiation of fumigation will be characterized by a lateral spread σ_{yi} and a vertical spread σ_{zi}; the initial plume height is h_i. Other parameters needed in scaling the results are the initial emission rate M_i of the fumigant (mass per unit time); the mean boundary-layer height during fumigation, $\bar{h}$; the mixed-layer convective velocity scale, w_*; the average wind speed U_i during fumigant emission at the effective stack height; the mean wind speed, U, in the mixed layer, and the angle, α, between its velocity and that just above the mixed layer.

One of the earliest treatments of fumigation into a mixed layer (Turner, 1969, p. 35) was based on the specification that the lateral spread at ground level, $\sigma_y(0)$, is given by

$$\sigma_y(0) = \sigma_{yi} + \bar{h}/8 \tag{1}$$

and that an increment of fumigant entering the mixed layer is perfectly mixed in the vertical by the time any of it appears at the surface. Lyons and Cole (1973) applied Turner's model to case (b), while allowing $\sigma_y(0)$ to increase with increasing downstream distance; Meroney *et al.* (1975) used a similar treatment. van Dop *et al.* (1979) also treated case (b) and utilized the mass diffusion equation in the lateral direction; they obtained maximum ground-level concentrations several times smaller than those of Lyons and Cole (1973) and Meroney *et al.* Misra (1980a, b) developed a model similar to that of van Dop *et al.* and presented a comparison against some observed data.

All of the above investigators assumed instantaneously perfect vertical mixing of entrained pollutants, and all adopted a model boundary layer whose top is a smoothly varying function of distance rather than one having locally turbulent protuberances whose peak-to-trough amplitudes could exceed several σ_{zi}. The need for improving these two assumptions,

and their qualitative effects upon ground-level concentrations, will become apparent later in this study.

2. LABORATORY PROCEDURES AND QUALITATIVE OBSERVATIONS

Our laboratory model utilizes a water convection tank of length 1.2 m on a side and mixed-layer depth in the vicinity of 0.2–0.3 m (see Willis and Deardorff, abbreviated W & D, 1974). An initial stably stratified layer is gradually consumed by a growing turbulent mixed layer generated by surface heating. It has previously been shown that the vertical and lateral dispersion velocities are similar to those in the idealized atmospheric mixed layer, for a mean wind not too strong, when turbulent velocities are scaled by w_*, lengths by $\bar{h}$ and eddy lifetimes by $\bar{h}/w_*$ (W & D, 1976). Experimental data on w_*, w_s, the lapse rate aloft, etc., were approximately as specified in Deardorff *et al.* (1980) for experiment E6 for slow entrainment rates, and E8 for fast entrainment.

The primary fumigating material used was a blue dye (orogliocine a supra) released slowly and steadily from a section of a small cylinder towed along a horizontal line in the middle of the tank at a height just above the uppermost reaches ($z = h_2$) of the existent mixed layer. The initial fumigant ribbon was about 0.8 cm in diameter, but was subject to local gravity-wave undulations of amplitude about ± 1 cm before fumigation began. The height h_2 is a measure of the maximum height of the penetrating mixed-layer thermals existing at any given time. It was determined experimentally by the height of disappearance of index-of-refraction variations experienced by a horizontally spread laser beam passing through the tank. The beam could be positioned vertically and its height monitored continuously while tracking $h_2(t)$. From the previous entrainment experiments cited it was known how far below h_2 the mean height $\bar{h}$ was—typically $0.13\bar{h}$ below. In this manner, $\bar{h}(t)$ was determined.

Dye concentrations were measured through use of a light extinction technique. A second laser beam of 1 mm diameter was directed horizontally underneath and along the initial fumigant axis at a height 2 cm above the surface (below this height there was too much scintillation to use this technique). The attenuation of light along this beam across the length of the tank, due to portions of descending dye intercepting the beam, was measured by means of a photocell receptor. Calibrations confirmed that for dye concentrations used in the experiments, the light absorption closely followed Beer's law. In each experiment an *in situ* calibration was obtained by measuring the light intensity shortly before fumigation had commenced, and immediately after the experiment when the total dye mass of dye emitted had been stirred thoroughly into the known volume of water in the tank.

In an additional set of 5 experiments with fast entrainment, nearly neutrally buoyant oil droplets were used as the fumigant. Photographs were analyzed which viewed the droplets in the y–z plane after summing them over x, the fumigant axis. Near-surface dimensionless concentrations underneath the fumigant axis were then obtained, along with $\sigma_y(0)$ values which could not be sampled well with the dye technique. This oil-droplet method was used previously for measuring concentrations from releases within the mixed layer (see W & D, 1976; 1978; 1981). However, it was not utilized here for obtaining concentrations in the case of slow entrainment because the droplets are subject to a slow buoyancy instability when suspended in a stably stratified layer (and awaiting fumigation). The coefficient of thermal expansion of an oil mixture increases more rapidly with temperature than does water, so that if a droplet initially at equilibrium in a quiescent environment gets perturbed downward (or upward), it eventually finds itself slightly heavier (or lighter) than the surrounding fluid. In the fast-entrainment case, the stratification in the stable layer was much reduced; this, along with the shorter period required for fumigation then, allowed us to use the droplet technique in this case.

In order to obtain w_*, for scaling purposes, the surface heat flux needs to be known approximately. The latter was obtained from thermocouple measurements of the rate at which the mixed layer was warming with time, assuming that 20 % of the warming was caused by entrainment. Resulting values of w_* averaged 0.9 cm s^{-1}.

The appearance of a dye streak before and during fumigation is shown in Figs 1(a)–(d). In Fig. 1(a) the unfumigated dye ribbon is seen to be undergoing gravity-wave undulations, while Fig. 1(b) depicts a classic entraining wisp of fumigant on the left. Figure 1(c) discloses fumigation occurring at numerous spots along the line. By the time of Fig. 1(d), about one-half the initial fumigant line has been entirely engulfed by the mixed layer while much of the other half still remains unfumigated. Visual observations and movies indicated that the clear areas where fumigation was first completed aloft were regions where the diverging tops of dominant thermal plumes swept the fumigant to the sides and downwards. Soon thereafter these portions of dye were entrained at locations considerably removed from the original locations.

Photographs such as Fig. 1(d) provide direct evidence that no matter how compact the initial fumigant plume may be, a rather long time, of order $(5–10)\bar{h}/w_*$ (depending on the entrainment rate), is required before all the fumigant material is entrained into the mixed layer. Thus, it is not only the finite value of σ_{zi} that causes the fumigant to be entrained somewhat gradually, rather it is the variability in local mixed-layer height which ranges from below $\bar{h}$ to h_2 at any given time at different locations. The same conclusion carries over to case (b) of the shoreline internal boundary layer, where, at a given point on the surface, the

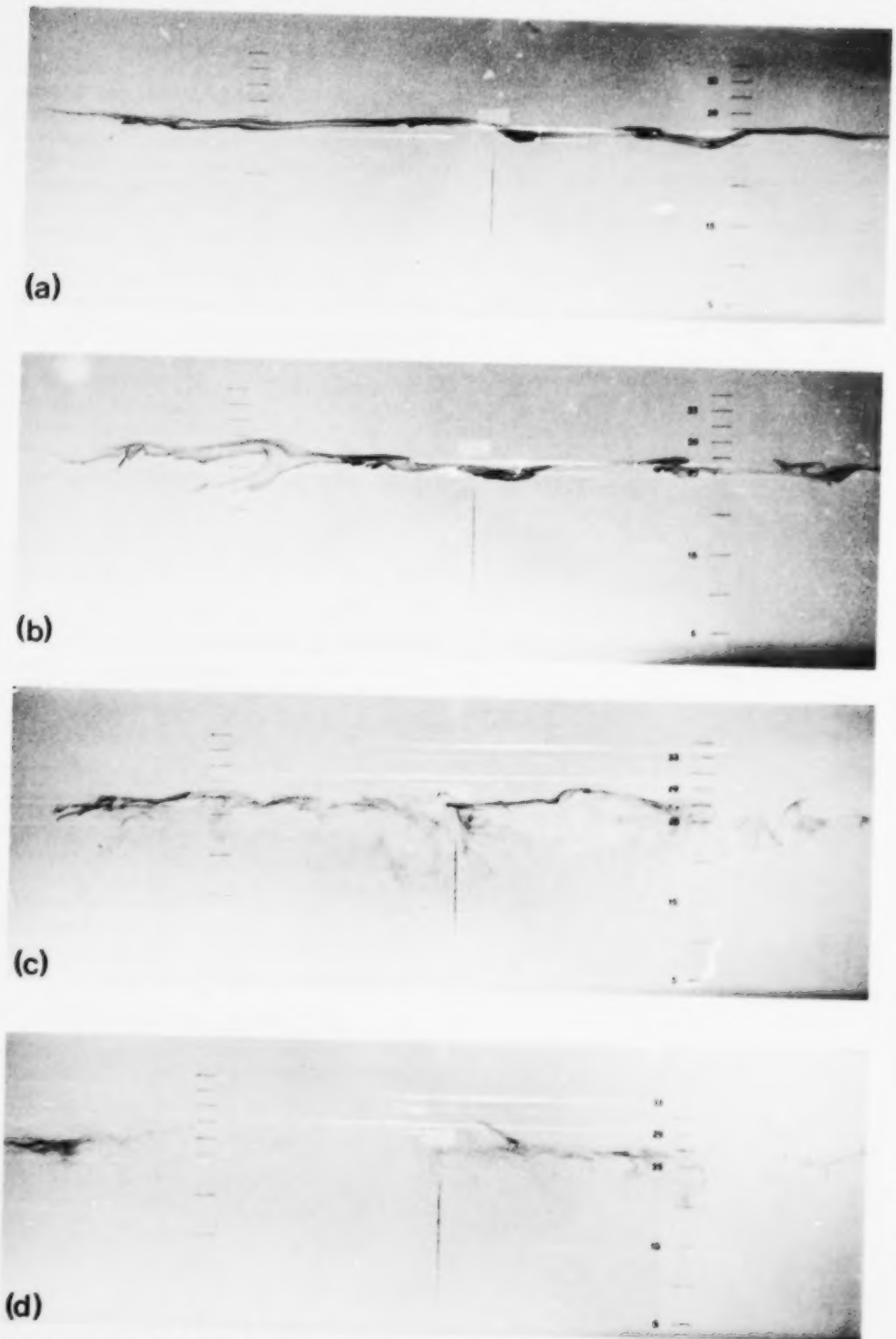

Fig. 1. Side view of a dye ribbon in a slow-entrainment experiment. Initially, $\bar{h}$ is about 21 cm; h_i is 26 cm. (a) $T = -2.6$ (before fumigation has commenced aloft); (b) $T = 2.7$ (shortly after trace amounts of fumigant were noticed near the surface); (c) $T = 6.4$ and (d) $T = 8.7$. The lower bright horizontal line is the position of the h_2-determining laser beam against the far side wall where a height scale (ticks on the right) is labelled 5, 15, 25, 29 and 33 cm. A second bright line above the first is a reflection.

local height of the mixed layer fluctuates in time over the same height range relative to $\bar{h}$, yielding a gradual consumption of a compact fumigant in the statistical mean.

3. QUANTITATIVE EXPERIMENTAL RESULTS

The case of slow entrainment rate ($w_e/w_* = 0.015 \pm 0.003$) will be treated first.

Slow entrainment

Values of dimensionless near-ground level concentration

$$C(0,0) \equiv U_i c(0,0)\bar{h}^2/M_i \tag{2}$$

obtained from the average of 5 experiments using the dye-absorption technique are shown in Fig. 2 as a function of dimensionless time T:

$$T \equiv tw_*/\bar{h}. \tag{3}$$

Here, $c(0,0)$ is the mean concentration near the surface (at $z = 0.07\bar{h}$) along the axis $y = 0$; the zero of the time axis is taken to be the earliest time at which the initial dye ribbon was observed to be subjected to (downward) diffusion motions. The time scale $\bar{h}/w_*$ in (3) has a characteristic value of 5 min. On the average, in all experiments, about 1.7 of these time units were required before the first traces of fumigant were sensed to have reached the near-surface level. This implies that the instantaneously well-mixed assumption for fumigant just entering the mixed layer could be seriously in error.

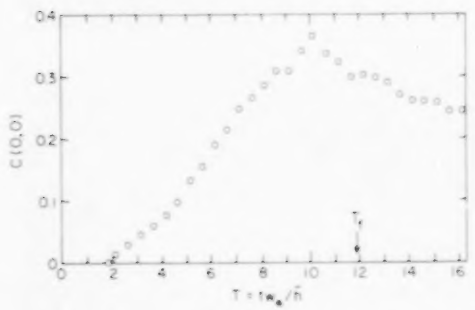

Fig. 2. The data of $C(0,0,T)$ from the average of 5 slow-entrainment experiments. The arrow indicates time when initial fumigant ribbon was no longer discernable anywhere.

The arrow at $T = 12 = T_f$ in Fig. 2 represents the average dimensionless time at which the last bit of the initial dye ribbon was visually estimated to have been fumigated into the mixed layer. The large value of T_f, and the fact discussed earlier that it is independent of σ_{zi} for $\sigma_{zi} \ll \bar{h}$, is perhaps the most surprising or important finding of this study. The expectation had been that, for $\sigma_{zi} \to 0$, the fumigation would occur everywhere very soon after it starts anywhere, in this case of horizontal homogenity in the mean. Instead, the variability of local mixed-layer height dominates the situation. C does not reach a maximum until shortly before T_f; namely at $T \cong 10$ in this case. Then it slowly drops off as very little additional fresh fumigant is available aloft but lateral spreading within the mixed layer continues.

The five experiments comprising Fig. 2 came from two sets: in one set of 3 the average mixed-layer height was 0.27 m, in the other set of 2 it was 0.18 m. Yet, the scaled data appear the same, within the scatter of sampling error, as shown in Fig. 3 which is a verification of mixed-layer scaling within the laboratory.

Fast entrainment

A plot of $C(0,0,T)$ for the fast-entrainment case ($w_e/w_* = 0.042 \pm 0.003$) is shown in Fig. 4. Also shown for comparison are the slow-entrainment results. The arrow denotes the time, T_f, at which the initial dye ribbon was estimated to have disappeared in the fast-entrainment case. Also shown are the oil-droplet data values which increase more rapidly with time than do the dye-technique data points although both apply to

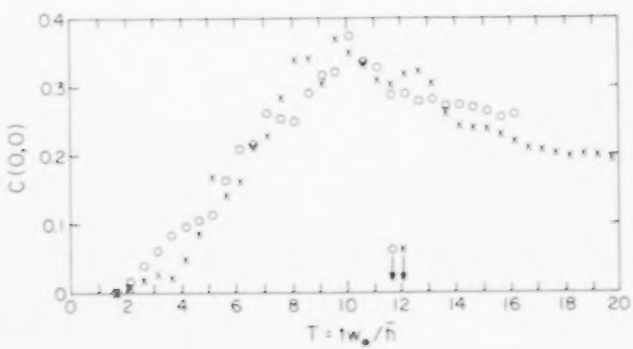

Fig. 3. Data points of $C(0,0,T)$ for the slow-entrainment case: from the average of 3 experiments with $\bar{h} = 0.27$ m (circles), and from the average of 2 experiments with $\bar{h} = 0.125$ m (x's). Explanation of arrows as in Fig. 2 caption.

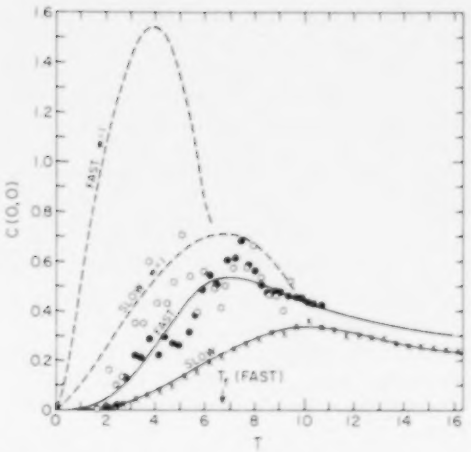

Fig. 4. The data on $C(0, 0, T)$ from the average of 2 fast-entrainment experiments using dye (solid circles), and from the average of 5 fast-entrainment experiments using oil droplets (open circles). Also shown are the slow-entrainment data (x). The curves, which represent a model with gradual (solid curve) and instantaneous (dashed curve) mixing of intercepted material through the mixed layer, are explained in Section 4.

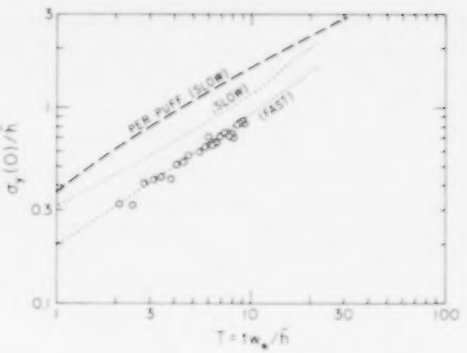

Fig. 5. Lateral spread at $0.07\bar{h}$ normalized by $\bar{h}$ as a function of dimensionless time T (measured from the onset of fumigation aloft). Oil-droplet data from the average of 5 experiments with fast entrainment are denoted by circles. The data are effective σ_y values. The curves are explained in Section 4.

this same case. We believe that most of the discrepancy lies in uncertainty of the time origin in the oil-droplet data, which was positioned by adding 1.7 time units to the last photograph not yet revealing the presence of any oil droplets near the surface. However, by the time the first droplet did appear there in any given realization, it is very likely that traces of dye would have been present at least 1 time unit earlier. Hence, we place greater credence on the dye-technique data.

The peak mean value ($C \cong 0.6$) is understandably greater than with the slower entrainment rate, since the same amount of fumigant is injected more quickly while the lateral spread within the mixed layer is smaller. The peak, therefore, occurs earlier, near $T = 7$. The curves in Fig. 4 will be discussed in Section 4.

Lateral spread

The near-surface standard deviation, $\sigma_y(0)$, normalized by $\bar{h}(t)$ and plotted against T, is shown in Fig. 5 for the fast-entrainment case. The main result here is that $\sigma_y(0)$ has a substantial value in the vicinity of $0.3\bar{h}$ at the earliest stage at which it can be estimated. This value is about 2.4 times larger than that given by $\bar{h}/8$ in (1). Significant lateral diffusion is observed to occur near $z = h_2$ at the onset of interception of a particular parcel of fumigant, so that $\sigma_y(\bar{h})$ becomes substantially larger than σ_{yi} even before fumigants can be noticed at ground level. Little if any variation of σ_y with height was observable during the experiments. However, if a particular entrained puff of fumigant could be followed as it mixes, its representative σ_y would be expected to increase with time as it descends on the average.

The $\sigma_y(0, 0)$ data of Fig. 5 therefore do not have the usual meaning of σ_y at a fixed distance downstream from a continuous source. Instead, they represent effective lateral spreads arising from a contaminant population possessing a wide range of diffusion travel times. That is, for $T = 7$, for example, the near-surface contaminants are comprised partly of fresh "puffs" with rather small $\sigma_y(0)$ that had entrained only about 2 units earlier, and partly of older puffs with substantially greater $\sigma_y(0)$ that had entrained up to 5 or 6 time units earlier. For this reason, in part, the $\sigma_y(0)/\bar{h}$ data values in Fig. 5 are smaller than found in previous studies with a unique travel-time origin of $T = 0$, an example of which is the dashed-curve parameterization of $\sigma_y(0)$ from an instantaneous near-surface source at $T = 0$ (labelled "per puff") from experiments with slow entrainment (Deardorff and Willis, 1975).

4. A MODEL INCORPORATING THE NEW FINDINGS

A new model of fumigation is required in order to take account of the dominant influence of the variability of the local mixed-layer height, and to remove the assumption of instantaneously perfect vertical mixing of entrained fumigants between $z = 0$ and $\bar{h}$. Here, a model is developed that estimates the $C(0, 0, T)$ data of Fig. 4 for the horizontally homogeneous case in the mean. Extension of such a model to the lakeshore fumigation case is discussed in Section 5.

The model development described in this section applies to the case of a fumigant plume that is initially compact vertically, so that σ_{zi} is irrelevant. Extension to the more general case is described in the Appendix.

As the mixed-layer top rises with time, (or as air above the mixed layer entrains into the latter), it will have intercepted and entrained some fraction $A(t)$ of the fumigant by the time t, where $0 \leq A \leq 1$. The fractional mass of fumigant per unit length intercepted in the time increment dt is $M_t(\partial A/\partial t)dt$. If this

increment were vertically well mixed, upon entering the mixed layer, its immediate contribution to the mean centerline ground-level concentration $c(0,0)$ would be

$$dc(0,0,t) = M_l \partial A/\partial t\, dt/[\sqrt{2\pi}\sigma_y(0)\bar{h}] \quad (4)$$

where $\sigma_y(0)$ here refers to the mean lateral spread of the increment very shortly after its fumigation, and M_l is the initial fumigant mass per unit length of plume. (The lateral spread has been assumed to be Gaussian.) Upon using the dimensionless variables, (4) becomes

$$dC(0,0,T) = \partial A/\partial T/[\sqrt{2\pi}\sigma_y(0)/\bar{h}]. \quad (5)$$

The net concentration at time T from all such increments that had fumigated up to that time, and each of which had commenced fumigation at the variable time T', is

$$C(0,0,T) = (2\pi)^{-1/2}\int_0^T [\partial A(T')/\partial T']/ [\sigma_y[T-T')/\bar{h}]dT'. \quad (6)$$

In this manner the larger values of $\sigma_y/\bar{h}$ from the earlier chunks of fumigant are taken into account.

However, the vertical mixing is by no means instantaneous, as pointed out earlier. Therefore, a weighting factor, $W(T-T')$, for each entrained increment is introduced into (6), where $0 \leqslant W \leqslant 1$, giving

$$C(0,0,T) = (2\pi)^{-1/2}\int_0^T [W(T-T')\partial A(T')/\partial T']/ [\sigma_y(T-T')/\bar{h}]dT'. \quad (7)$$

The weighting factor, W, is parameterized by

$$W(T-T') = 3T''^2 - 2T''^3 \quad (T'' < 1)$$
$$= 1 \quad (T'' \geqslant 1) \quad (8)$$

where

$$T'' \equiv (T-T')/4. \quad (9)$$

Equation (8) is the simplest cubic polynomial having zero slopes at $T'' = 0$ and 1. With the constant specified as 4 in the denominator of (9), only 16 % of the well mixed value from any particular entrained parcel of fumigant is prescribed to appear at ground level one time unit later, while 84 % of its well mixed value appears there 3 time units after its entrainment and 100 % by 4 time units. Previous studies by the authors (W & D, 1976) have indicated that a dimensionless time of 3–4 units is necessary before vertical mixing is nearly complete from an instantaneous line source within the mixed layer.

The entrained fraction $A(T')$ in (7) is similarly parameterized by

$$A(T') = 3(T'/T_f)^2 - 2(T'/T_f)^3 \quad (T'/T_f < 1)$$
$$= 1 \quad (T'/T_f \geqslant 1) \quad (10)$$

and T_f is the total dimensionless period of fumigant capture by the mixed layer, in this situation of horizontal homogeneity in the mean. Thus, as the uppermost reaches of the mixed layer first begin to intercept fumigant, the rate of interception is very small; similarly when fumigation aloft is nearly complete. The delay after fumigation first commences aloft and fumigant is first noted at the surface, is thus a product of the slow initial rise of the fumigant entrained fraction A, and of the vertical-mixing weighting factor, W (when $T' = 0$ in (8) and (9)).

The fumigation period T_f is parameterized by

$$T_f = 0.42(\Delta h/\bar{h})/(w_e/w_*) + 2.3 \quad (11)$$

where $\Delta h = h_2 - h_0$ is the full vertical extent of the zone of variable mixed-layer height; i.e., below $z = h_0$ the air can be considered well mixed in potential temperature. Now, if the irregular mixed-layer top were frozen in the horizontal while h_2 and h_0 advanced upwards at the rate w_e, one would expect $T_f = (w_*/\bar{h})\Delta h/w_e$. However, due to the motions and varying sizes of the dominant thermal eddies, and due to their growth and decay, the preceding expression is longer than that which is observed and parameterized by (11). The two constants in (11) were deduced from the positions of the two data peaks for slow and fast entrainment in Fig. 4. Equation (11) yields the observed value of T_f in the slow-entrainment case, but underpredicts it by 13 % in the fast-entrainment case. A minimum value for T_f, which is associated with a maximum in w_e, is sometimes attained during the late morning "explosive growth" period. The maximum in w_e/w_* has been measured to be 0.24 in the laboratory by Deardorff *et al.* (1980). In the same study the dependence of $\Delta h/\bar{h}$ on an overall Richardson number is presented, from which the following parameterization may be deduced:

$$\Delta h/\bar{h} = 0.2 + 4w_e/w_*.$$

The following parameterization for $\sigma_y(T-T')$ in (7) from Deardorff and Willis (1975) is used:

$$[\sigma_y(T-T')/\bar{h}]^2 = [B(w_e/w_*)]^2\, 0.26(T-T')^2/ [1+0.91(T-T')] + (\sigma_{yi}/\bar{h})^2 \quad (12)$$

except that they did not include the entrainment-rate dependence of $B(w_e/w_*)$. Thus, when σ_{yi} is small, σ_y from (12) varies as $T-T'$ at small times following entrainment of the parcel, and as $(T-T')^{1/2}$ at large times.

The prescribed dependence of B upon w_e/w_* is

$$B = 1.3/(1 + 20w_e/w_*). \quad (13)$$

This tentative dependence is derived from the study of W & D (1974) where about 25 % greater values of horizontal velocity variance within the mixed layer were obtained when $w_e/w_* = 0.013$ than when it was 0.023. Assuming σ_y is reduced in the case of more rapid entrainment by the square root of the reduction in velocity variance, a two-parameter formulation like (13) results. For the case of slow entrainment, B in (13) becomes unity.

The physical mechanism which (12) and (13) seek to represent is the effect of non-turbulent air, carrying fumigant, being entrained into the mixed-layer where it acquires vorticity and diffusive capability only gradually. Furthermore, as this air enters the mixed layer, it tends to bring with it a mean horizontal momentum (zero in the convection tank) which must be accelerated by the mixed-layer turbulence before it attains the local value possessed by the latter. In so doing, the turbulence within the mixed layer must be damped increasingly with increased entrainment rate, thereby providing an explanation for the difference in turbulent-velocity curves in W & D (1974, Fig. 6) which at the time was ignored. In developing the present model, however, it was noticed that if this effect were omitted, no satisfactory agreement with the present fumigation data could be attained.

Using (7) and subsequent equations, and a time step of $dT' = 0.01T_f$, the (solid) $C(0,0,T)$ curves of Fig. 4 for the two cases of slow and fast entrainment were obtained numerically. In (11), $\Delta h/\bar{h}$ was taken as 0.27 and 0.35, respectively. Because of the four empirical constants [1 in (9), 2 in (11), and 1 of the 2 in (13)] which are free to be altered by about $\pm 30\%$ each, the model curves can be considered to be a parameterization of the data, albeit a parameterization which attempts to take into account the more important physical processes associated with entrainment.

The two dashed curves in Fig. 4 were obtained from the model by setting $W = 1$ in (7) while holding all else the same, i.e., assuming instantaneously perfect vertical mixing of air entraining into the mixed layer as is assumed by existing fumigation models. The huge discrepancy which this assumption causes should not be too surprising, since the need for gradual vertical diffusion is well known in calculations of concentrations downstream from a near-surface source.

The present model has been used to generate $C(0,0,T)$ curves for the several values of w_e/w_* indicated in Fig. 6. The curves have the same general shape as those predicted by the model of Misra (1980b). In that model the initial vertical spread of the fumigant in a qualitative sense can substitute for omitting the effect of local variations in mixed-layer height. In the Appendix it is shown that only for $\sigma_{zi}/\Delta h$ exceeding about 0.5 does the effect of the initial vertical spread dominate over that of finite Δh in causing $C(0,0,T)$ to vary only gradually in time.

The strong dependence of the peak mean ground-level concentration and its time of occurrence on w_e/w_* in Fig. 6 points out a need for the capability of predicting w_e. If the potential temperature change, $\Delta\Theta$, across the h_0 to h_2 "capping inversion" region is unknown, so that an overall Richardson-number approach (e.g., see Deardorff *et al.*, 1980) is inapplicable, the next best way to estimate w_e is probably through an encroachment-type formula (Carson and Smith, 1975). That method, which results when $\Delta\Theta$ is assumed constant in time, gives

$$w_e = w_*^3/[(g/\bar{\Theta})\bar{h}^2\gamma)] \tag{14}$$

where γ is the value of $\partial\bar{\Theta}/\partial z$ just above the entrainment zone or capping inversion.

The present model has also been utilized to reconstruct the effective near-surface σ_y values, $(\sigma_y)_{\text{eff}}$, during fumigation, given the $\sigma_y/\bar{h}$ formula of (12) which applies "per puff". The equation connecting the two is

$$(\sigma_y)_{\text{eff}}^2(T) = \int_0^T \sigma_y^3(T-T')(\partial C/\partial T')\,dT' \Big/ \int_0^T \sigma_y(T-T')(\partial C/\partial T')\,dT' \tag{15}$$

where σ_y is the lateral spread following any particular puff originating at time T'. For the case of fast entrainment, the lower dotted curve of Fig. 5 was obtained, which agrees well with the observed data for

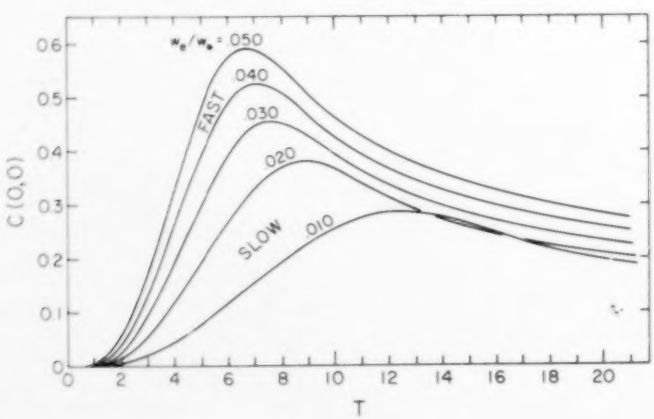

Fig. 6. Model derived curves of $C(0,0)$ as a function of T for dimensionless entrainment rates different than the two studied experimentally.

$(\sigma_y)_{\text{eff}}$ for the fast-entrainment case. This provides an independent check on (12) and (13) which were utilized in the model. For the case of slow entrainment, (12) and (13) produce the upper (dashed) curve of Fig. 5 for a puff entraining at $T = 0$. Utilization of (15) by the model for $w_e/w_* = 0.015$ then produced the dotted $(\sigma_y)_{\text{eff}}$ curve labelled "slow" in Fig. 5, with its greater lateral spread than that of the fast-entrainment case.

The effective σ_y curves of Fig. 5 (dotted) show a slight upturn at about the time that fumigation is complete. The upturn at this time is caused by the fact that entraining parcels with small σ_y values are no longer being averaged in with earlier parcels that have greater σ_y values.

In following sections it will be assumed that $(\sigma_y)_{\text{eff}}$ is the lateral spread of a Gaussian distribution at any time T. However, if the Gaussian assumption is true for parcels all having the same travel time, T, it is not necessarily true for the aggregate concentration from parcels possessing distinctly different travel times. Nevertheless, any deviation from gaussianity for the effective second moment in y is probably small because the entraining parcels of fumigant have a smooth, continuous distribution of starting times and engulfment rates, in the statistical mean.

5. APPLICATION TO FUMIGATION AT LAKESHORE

The preceding results can be applied to the lakeshore fumigation problem in the following manner. In Fig. 7 the irregular entrainment interface $h(x, t)$ is sketched between the h_0 and h_2 curves which mark its envelope. The mean value of $h(x, t)$ is denoted $\overline{h}(x)$. The point x_0 depicts where fumigants first start to entrain aloft, on the average, as a result of penetrating plumes occasionally reaching $z = h_2$. Taylor's translation hypothesis is then applied, as customary, to relate diffusion travel time, t, to downstream distance, x:

$$x - x_0 = U(t - t_0) \tag{16}$$

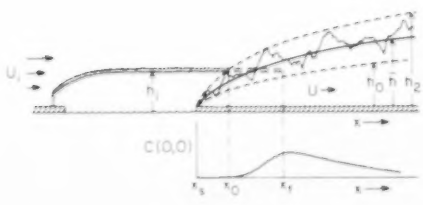

Fig. 7. Illustration of the shoreline fumigation situation. A compact smoke plume from a stack at effective height h_i drifts above shallow boundary layer over lake. Its interception by the mixed-layer over heated ground does not commence until $h_2(x_0) = h_i$, and is not completed aloft until a distance $x_f - x_0$ further. A sketch of the dimensionless surface concentration along the fumigant axis, $C(0, 0)$, as a function of x is shown below the internal boundary layer.

or, in terms of a dimensionless downstream coordinate, X, (16) becomes

$$X \equiv [(x - x_0)/\overline{h}](w_*/U) = T. \tag{17}$$

This analogy makes use of the fact that the entrainment rate is essentially the same, for given external conditions, whether the top of the mixed layer is horizontally homogeneous in the mean, or is sloping as with the internal boundary layer shown in Fig. 7. It should be noted that the slope is greatly exaggerated in the sketch, being on the order, typically, of 0.01 or less when $\overline{h}$ exceeds a few hundred meters. Also, w_* is assumed to be approximately independent of $x - x_0$ since the variations in $\overline{h}$ and surface heat flux tend to compensate, and since the dependence on either is only to the 1/3 power.

Thus, the concentration curves of Figs 4 and 6 are expected to apply closely to the lakeshore fumigation situation if T is replaced by X from (17). The dimensionless concentrations then represent time-mean values at fixed points along X. The averaging period corresponds to the average life-time of the longer-lived boundary-layer scale eddies, estimated to be 5 to $10\overline{h}/w_*$, or typically, 15–30 min. If the averaging time in the atmospheric mixed-layer is sufficiently long to encompass periods when mesoscale or synoptic scale variations in wind velocity occur, a smaller mean maximum is to be expected than those given in Figs 4 and 6 because only turbulence of boundary-layer scale and smaller is present to disperse the fumigants in the laboratory convection tank.

There is one distinct difference between this situation and that of the horizontally homogeneous case, however. Because of the presence of downstream diffusion in the lakeshore situation, but not in the horizontally homogeneous case, the present results are upper limits when applied to the lakeshore fumigation problem.

To obtain the distance from the coastline to x_0 in Fig. 7 requires adopting a model of the mixed-layer growth for $h_2(x - x_s)$, where x_s is the value of x at the shoreline. One such model is that of Venkatram (1977) for which $\overline{h}(x - x_s)$ is proportional to $(x - x_s)^{1/2}$. Then x_0 can be determined by the interception of $h_2 \cong 1.15\overline{h}$ with the initial plume height, h_i.

6. TIME-AVERAGED CONCENTRATIONS

The time-averaged concentration is of interest, as it represents the dosage that can be experienced at a fixed point. In the lakeshore fumigation situation, results already represent time-averaged values, for a continuous upwind source. However, in the horizontally homogeneous case, the fumigant eventually disappears and the time-averaged concentrations will depend explicitly on the averaging time. They will also depend on the angle, α, between the mixed-layer wind velocity and the fumigant-plume x axis. The simplest case of $\alpha = 0$ will be treated first.

The case $\alpha = 0$

The average ground-level concentration experienced at some point directly underneath the x axis can in this case be obtained simply by averaging the curves of Figs 4 or 6 over the desired dimensionless averaging interval, $\tau w_*/\bar{h}$. By testing different values obtained when a given averaging period is centered at different times, the maximum time-averaged ground-level concentration can be selected. This maximum value, again dimensionless, is shown as a function of $\tau w_*/\bar{h}$ in Fig. 8 (upper curve) for the case of $w_e/w_* = 0.015$ (slow entrainment). A typical value for the maximum one-hour average, for $\tau w_*/\bar{h} = 3600\,\text{s} \times 1.5\,\text{m s}^{-1}/500\,\text{m} = 10.8$, is $C_{max} = 0.28$. This value is a surprisingly large fraction of the maximum expected value (0.34) without any time averaging. It should be kept in mind, of course, that local *peak* values of concentration will be much greater than either of these values, in any particular realization or event.

For the case of "fast" entrainment, $C_{max}(\tau)$ is shown in Fig. 9 (upper curve). In obtaining these curves, the $C(T)$ data from the model of Section 4 were utilized. The other curves in Figs 8 and 9 will be explained next.

The case $\alpha \neq 0$

In this case abrupt wind shear exists at the top of the mixed layer, over the angle α. The source line will be

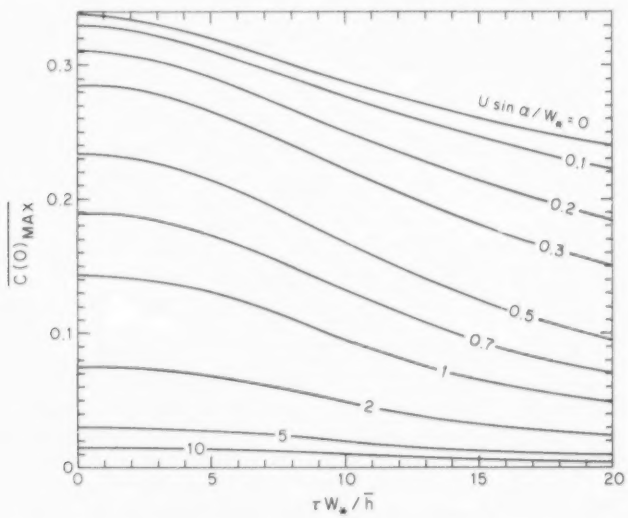

Fig. 8. A plot of $(\overline{C})_{max}$ near ground level obtained by averaging $C(0, 0)$ over the averaging period $\tau w_*/\bar{h}$ and selecting the maximum; it is plotted against the dimensionless averaging period. Case of slow entrainment. Different curves are for the different values of $U \sin|\alpha|/w_*$.

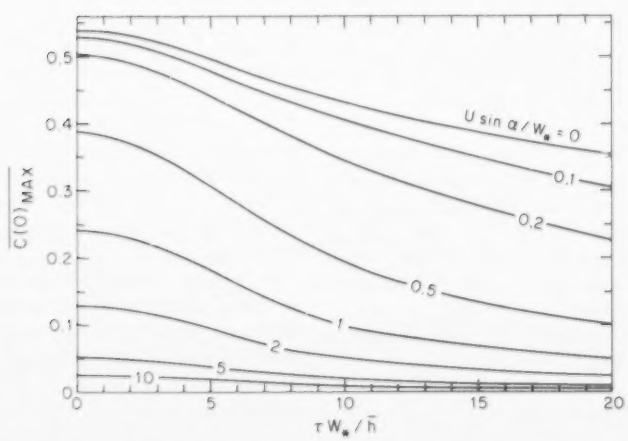

Fig. 9. A plot of $(\overline{C})_{max}$ vs $\tau w_*/\bar{h}$ as in Fig. 8, except for the case of fast entrainment.

assumed to move parallel to itself, which is a reasonable assumption if not too much time has elapsed between emission and fumigation. Then, the mixed layer will have a translation speed of $U \sin \alpha$ in a direction normal to the fumigant axis, where U is the mean wind speed within the mixed layer. This component advects each increment of fumigating material away from the fumigant line, thereby decreasing the ground-level concentrations, even at the point of maximum concentration, from what they were in the previous case with $\alpha = 0$.

A method by which this effect can be quantified has been suggested by G. A. Briggs (personal communication). That is to utilize the model of Section 4 to calculate the ground-level concentration at some fixed point lying a distance D normal to and downstream of the surface projection of the fumigant axis. Upon assuming a Gaussian lateral spread as before, (7) is then extended to

$$C(D,0,T) = (2\pi)^{-1/2} \int_0^T \{G[\,W(T-T')\partial A(T')/\partial T'\,]/ [\sigma_y(T-T')/\overline{h}\,]\}\,\mathrm{d}T' \tag{18}$$

where G is the Gaussian function

$$G = \exp\{-[D/\overline{h} - (U \sin\alpha/w_*)(T-T')]^2/ [2(\sigma_y(T-T')/\overline{h})^2]\}. \tag{19}$$

A particular value of $D/\overline{h}$ in (19) exists for which the ground-level concentration will be a maximum, and for a somewhat different value of $D/\overline{h}$ the time-averaged concentration will be maximized. By trial and error it was found that these maxima occurred for $D/\overline{h}$ in the range of 0.45–0.55 of $(U \sin\alpha/w_*)T_f$ in the case of slow entrainment, and in the range of 0.6–0.9 of this quantity in the case of fast entrainment. The corresponding curves of the maximum time-averaged ground-level concentration as a function of averaging period are shown in Figs 8 and 9 for the two respective cases, both of which refer to the situation of the horizontally homogeneous (in the mean) mixed layer. The effect of wind shear at the top of the mixed layer upon the maximum expected concentration in the steady shoreline fumigation problem is presumably obtained from Figs 8 and 9 for $\tau w_*/\overline{h} = 0$.

The fall-off in maximum time-averaged concentration as $U \sin|\alpha|$ increases is very pronounced, and indicates that the influence of a mixed-layer wind occurring at an angle to the initial fumigant axis should not be ignored, if possible. For $|\alpha| = 30°$, for example, $U = 4\,\mathrm{m\,s^{-1}}$ and $w_* = 2\,\mathrm{m\,s^{-1}}$, the quantity $U \sin|\alpha|/w_*$ is 1. In that case, with $\tau w_*/\overline{h} = 10$, the maximum ground-level dosage is only 33% of what it would have been with $\alpha = 0$, in the slow-entrainment case, and 23% in the fast entrainment case.

7. DISCUSSION

The maximum expected dimensionless ground-level concentration found here for the case of fumigation, before any time averaging is performed on the mean data, is 0.34 for slow entrainment and about 0.54 for fast entrainment. This contrasts with a value of about 3 found by W & D (1981) when the release was within the mixed layer at the height $h_i = 0.5\overline{h}$, and a value of about 6 when $h_i/\overline{h} = 0.24$ (W & D, 1978). Corresponding dimensionless ground-level concentration maxima obtained numerically by Lamb (1979) are not too dissimilar: 1.7 for $h_i/\overline{h} = 0.75$, 2.1 for $h_i/\overline{h} = 0.5$ and 4.5 for $h_i/\overline{h} = 0.25$. Thus, the effect of the fumigant being intercepted spottily by the mixed layer during the latter's growth or entrainment, instead of continuously as for a source within the mixed layer, plus the effect of contaminant injection at a maximum distance from the ground, combine to produce a relatively reduced maximum mean concentration. Temporal averaging and wind shear at the top of the mixed layer further reduce this maximum, for the horizontally homogeneous case, as in Figs 8 and 9.

These results apply most closely for the case $\sigma_{yi} \ll h_i$ and $\sigma_{zi} \ll h_i$. The study of Misra (1980a) indicates that in cases involving effluent travel times of about $\frac{1}{2}$ hour or more these conditions are not likely to be met. If σ_{yi}/h_i then is not small but σ_{zi}/h_i is, then the model of Section 4 can still be utilized to generate the $C(T)$ or $C(X)$ curve. However, if σ_{zi}/h_i is not small, then the present model should be extended as indicated in the Appendix.

The effect of wind shear at the top of the mixed layer is approximately taken into account by the analysis of Section 6 ($\alpha \neq 0$) which applies to abrupt lateral wind shear at $z = \overline{h}$ and little shear elsewhere. However, if the magnitude of the velocity jump at the top of the mixed layer, relative to w_*, becomes substantial (probably in the vicinity of 2 or more) the entrainment rate may be more rapid than that given by (14).

The authors have previously emphasized that the use of mixed-layer scaling for effluents released in the lower mixed layer begins to lose validity if U/w_* exceeds a value near 5–7, due to the importance then of mechanical mixing in the lowest 10% or more of the mixed layer. However, this limitation might be relaxed somewhat in the fumigation problem since turbulence in the upper mixed layer, which is most easily dominated by the thermal convection, then assumes more importance. Indeed, Misra (1980b) used convective mixed-layer scaling when U/w_* was as large as 12.

It is worthwhile summarizing the reasons why the $C(T)$ or $C(X)$ curves found in the present study may be upper limits to those occurring in the atmospheric mixed layer:

(1) Effects of mesoscale and synoptic-scale eddies or disturbances were not modeled. These, when important, delay the onset of the large-time lateral spread regime ($\sigma_y \propto t^{1/2}$) and thereby reduce the mean concentrations somewhat by the plume meander mechanism.

(2) The concentration data of Fig. 4 apply to the case of σ_{yi} and σ_{zi} small in comparison with h_i. When not small, the increased initial spreads, especially σ_{zi},

cause a reduction of maximum concentration after fumigation.

(3) In this study the concentrations were measured at the height $0.07\overline{h}$. Some slight reduction in values would be expected if they could have been measured at $z = 0$.

(4) In our laboratory study of the horizontally homogeneous mixed layer, in the mean, there was of course no downstream diffusion. When the present results are applied to the case of shoreline fumigation, replacing T with X, some degree of overprediction of concentrations should be expected.

(5) If the present results are utilized when U/w_* exceeds a value near 5–7, the additional shear-generated turbulence in the lower part of the boundary layer is expected to increase $\sigma_y/\overline{h}$ beyond the values observed in the laboratory free-convection mixed layer.

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APPENDIX: EFFECT OF σ_{zi}

The model of Section 4 includes the influence of σ_{yi}, but not of σ_{zi}. The latter effect can be included rigorously only by superposing the solutions $C(0, 0, T - T_0)$ for which the initial time, T_0, is allowed to vary instead of being held fixed at zero. Assuming the initial vertical spread is Gaussian, so that the solutions to be superposed are given Gaussian weighting with maximum weight at $T_0 = 0$, (7) becomes modified to

$$C(0,0,T) = (2\pi S)^{-1} \int_{-3S}^{3S} \exp[-\tfrac{1}{2}(T_0/S)^2] \int_{T_0}^{T} [\, W(T-T')\partial A(T')/\partial T' \,]/ [\sigma_y(T-T')/\overline{h}\,]\, dT'\, dT_0. \quad \text{(A1)}$$

In (A1), the superposition of concentrations is performed for T_0 varying from $-3S$ to $3S$, where S is the mean dimensionless time required for one vertical standard deviation, σ_{zi}, of the fumigant to be entrained:

$$S = (\sigma_{zi}/w_e)/(w_*/\overline{h}). \quad \text{(A2)}$$

In applying (A1) to some cases with finite σ_{zi}, the outer integral was approximated by subdividing the Gaussian distribution into 21 equal intervals between $\pm 3S$. Also, in order to specify σ_{yi} reasonably, it was assumed that

$$\sigma_{yi} = 2\sigma_{zi}. \quad \text{(A3)}$$

The study of Misra (1980a) suggests the factor of 2 in (A3), although a larger factor may apply for large travel times before fumigation.

The results utilizing (A1)–(A3) for various values of $\sigma_{zi}/\Delta h$ for the slow-entrainment case ($w_e/w_* = 0.015$) are shown in Fig. A1. In this figure, $T = 0$ still refers to the earliest time of interception by the mixed-layer top of fumigant if it had all been concentrated at the height $z = h_i$.

The case $\sigma_{zi}/\Delta h = 0.032$ applies approximately to the initial vertical spread in the laboratory ($\sigma_{zi} \approx 2$ mm, $\Delta h = 6.2$ cm). The dotted curve for this case shows only an insignificant deviation from, or smoothing of, the (solid) $\sigma_{zi} = 0$ curve. This result provides justification for the assertion that the laboratory experiments corresponded to cases for which the initial vertical spread was essentially zero.

As $\sigma_{zi}/\Delta h$ increases, the maximum mean ground level concentration can be noticed to decrease, and the time of maximum to extend to larger times following the first occurrence of significant fumigants at the surface. For $\sigma_{zi}/\Delta h$ as great as, or exceeding, 0.5, the effect of σ_{zi} in prolonging the fumigation episode and reducing the peak mean value appears to dominate over the similar effect of the stochastically varying mixed-layer height (i.e., the Δh effect).

The effect of σ_{yi} in moderating the fumigation was found to be very much smaller than that of σ_{zi} in all the cases examined, for $\sigma_{yi} = 2\sigma_{zi}$.

A convenient means of approximating the results from (A1) while using the simple model of Section 4 has been pointed out by H. A. Panofsky (verbal communication). The method is to increase the period of fumigant entry into the mixed

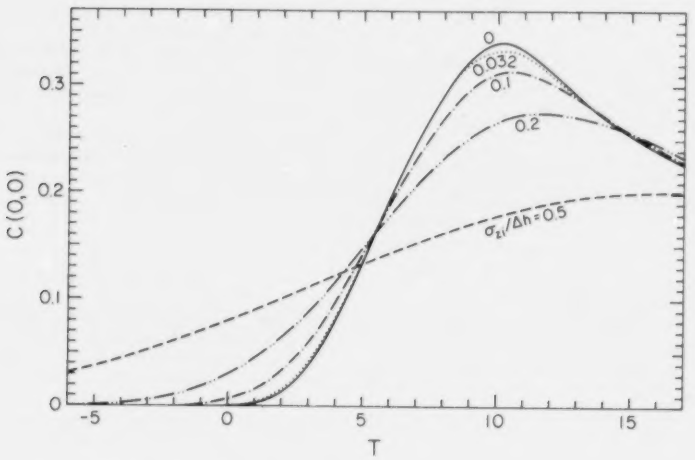

Fig. A1. Model curves of $C(0, 0)$ vs T for the case of slow entrainment, utilizing (A1)–(A3), for the designated values of $\sigma_{zi}/\Delta h$. The curve labelled "0" is also the lower curve in Fig. 4.

layer, t_f, by an amount proportional to σ_{zi}/w_e. A satisfactory expression for the effective dimensionless fumigant-entry period, T_{fe}, is found to be

$$T_{fe} = [T_f^2 + (4(\sigma_{zi}/\overline{h})(w_*/w_e))^2]^{1/2} \qquad \text{(A4)}$$

where T_f is still given by (11). The two contributions to T_{fe} are summed after the manner of variances from independent normal distributions, since the initial vertical distribution of fumigant is independent of the vertical distribution of mixed-layer air in the Δh region.

Thus, T_{fe} from (A4) may be used in place of T_f in (10) to allow for the effect of σ_{zi}, with (7) then being used as before to obtain the surface concentration. This method is found to yield accuracies within $\pm 6\%$ of the maximum mean concentration, in comparison with results of Fig. A1. Discrepancies are probably associated with small differences between the cubic-polynomial distribution for $A(T')$ in (10) and the cumulative Gaussian distribution. In using (A4), the initial time at which fumigant first enters the mixed layer is advanced from $T_0 = 0$ for $\sigma_{zi} = 0$ to about $T_0 = -1.2(\sigma_{zi}/\overline{h})/(w_e/w_*)$.